# Math 141  Section 2.1
## Example of Gauss-Jordan Elimination

**The original equations:**

$$\begin{aligned} 2x + 3y &= 7 \\ -x + 4y &= 2 \end{aligned}$$

**The original matrix:**

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ -1 & 4 & 2 \end{array}\right]$$

**Multiply Eqn 2 by 2:**

$$\begin{aligned} 2x + 3y &= 7 \\ -2x + 8y &= 4 \end{aligned}$$

**Multiply Row 2 by 2:**

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ -2 & 8 & 4 \end{array}\right]$$

**Add Eqn 1 to Eqn 2:**

$$\begin{aligned} 2x + 3y &= 7 \\ 11y &= 11 \end{aligned}$$

**Add Row 1 to Row 2:**

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 11 & 11 \end{array}\right]$$

**Divide Eqn 2 by 11 (multiply by $\frac{1}{11}$):**

$$\begin{aligned} 2x + 3y &= 7 \\ y &= 1 \end{aligned}$$

**Divide Row 2 by 11 (multiply by $\frac{1}{11}$):**

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 1 & 1 \end{array}\right]$$

**Multiply Eqn 2 by –3:**

$$\begin{aligned} 2x + 3y &= 7 \\ -3y &= -3 \end{aligned}$$

**Multiply Row 2 by –3:**

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & -3 & -3 \end{array}\right]$$

**Add Eqn 2 to Eqn 1:**

$$\begin{aligned} 2x &= 4 \\ -3y &= -3 \end{aligned}$$

**Add Row 2 to Row 1:**

$$\left[\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & -3 & -3 \end{array}\right]$$

**Divide Eqn 1 by 2 and Eqn 2 by –3:**

$$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$$

**Divide Row 1 by 2 and Row 2 by –3:**

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

# Example of Gauss-Jordan Elimination *with Pivoting*

**Initial matrix**

$$\left[\begin{array}{rrr|r} 2 & 4 & -2 & 12 \\ 3 & 5 & 0 & 13 \\ 1 & -2 & 1 & -4 \end{array}\right]$$

**Initial equations**

$$\begin{array}{rcrcrcr} 2x & + & 4y & - & 2z & = & 12 \\ 3x & + & 5y & & & = & 13 \\ x & - & 2y & + & z & = & -4 \end{array}$$

**$\frac{1}{2}\cdot$ Row 1**

$$\left[\begin{array}{rrr|r} \boxed{1} & 2 & -1 & 6 \\ 3 & 5 & 0 & 13 \\ 1 & -2 & 1 & -4 \end{array}\right]$$

**$\frac{1}{2}\cdot$ Eqn 1**

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 6 \\ 3x & + & 5y & & & = & 13 \\ x & - & 2y & + & z & = & -4 \end{array}$$

**Row 2 + (–3) · Row 1**
**Row 3 + (–1) · Row 1**

$$\left[\begin{array}{rrr|r} \boxed{1} & 2 & -1 & 6 \\ 0 & -1 & 3 & -5 \\ 0 & -4 & 2 & -10 \end{array}\right]$$

**Eqn 2 + (–3) · Eqn 1**
**Eqn 3 + (–1) · Eqn 1**

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 6 \\ & - & y & + & 3z & = & -5 \\ & - & 4y & + & 2z & = & -10 \end{array}$$

**(–1) · Row 2**

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 6 \\ 0 & \boxed{1} & -3 & 5 \\ 0 & -4 & 2 & -10 \end{array}\right]$$

**(–1) · Eqn 2**

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 6 \\ & & y & - & 3z & = & 5 \\ & - & 4y & + & 2z & = & -10 \end{array}$$

**Row 1 + (–2) · Row 2**
**Row 3 + (+4) · Row 2**

$$\left[\begin{array}{rrr|r} 1 & 0 & 5 & -4 \\ 0 & \boxed{1} & -3 & 5 \\ 0 & 0 & -10 & 10 \end{array}\right]$$

**Eqn 1 + (–2) · Eqn 2**
**Eqn 3 + (+4) · Eqn 2**

$$\begin{array}{rcrcrcr} x & & & + & 5z & = & -4 \\ & & y & - & 3z & = & 5 \\ & & & - & 10z & = & 10 \end{array}$$

**$-\frac{1}{10}\cdot$ Row 3**

$$\left[\begin{array}{rrr|r} 1 & 0 & 5 & -4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & \boxed{1} & -1 \end{array}\right]$$

**$-\frac{1}{10}\cdot$ Eqn 3**

$$\begin{array}{rcrcrcr} x & & & + & 5z & = & -4 \\ & & y & - & 3z & = & 5 \\ & & & & z & = & -1 \end{array}$$

**Row 1 + (–5) · Row 3**
**Row 2 + 3 · Row 3**

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \boxed{1} & -1 \end{array}\right]$$

**Eqn 1 + (–5) · Eqn 3**
**Eqn 2 + 3 · Eqn 3**

$$\begin{array}{rcrcrcr} x & & & & & = & 1 \\ & & y & & & = & 2 \\ & & & & z & = & -1 \end{array}$$